AN-8 Analysis of Piezoelectric Ceramics

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Introduction

This application note discusses the characterization and analysis of piezoelectric ceramics. We refer to lead zirconate titanate (PZT) to illustrate some of the concepts since it is the most widely used and studied piezoelectric ceramic. In this Application Note we attempt to cover the most common measurement methods as well as discuss parameters of interest. To thoroughly grasp the behavior of a piezoelectric polycrystalline ceramic, a basic understanding of the ceramic itself should not be overlooked. Several sources for more details of piezoelectric polycrystalline ceramics can be found in the Bibliography.

Piezoelectric Parameters

The piezoelectric parameters that are of interest when considering the electromechanical effects of piezoelectric materials are the piezoelectric voltage coefficients (*g31* and *g33*), the piezoelectric coupling factors (*k31*, *k33*, *kp* and *kt*) and the piezoelectric charge coefficients (*d31* and *d33*).

The *d*-coefficient is the constant between electric displacement and stress, or strain and electric field. High *d*-coefficients are desirable in materials utilized as actuators, such as in motion and vibration applications. High *g*-coefficients are desirable in materials intended to be used as sensors, to produce voltage in response to mechanical stress. The *g*-coefficient is related to the *d*-coefficient by the following expression:

$$d_{mi} = \sum\nolimits_{nm}^{T} g_{ni} \tag{1}$$

Where:

m, n =1, 2, 3 and i = 1, 2, ...6.

High *g*-coefficients are desirable in materials intended to be used as sensors, to produce voltage in response to mechanical stress. The piezoelectric coupling factor *k* is a measurement of the overall strength of the electro-mechanical effect. It is often defined as the square root of the mechanical energy available to the total electrical energy input. The value of *k* is of course always less than unity because there is always energy lost in the conversion.

Other important properties of PZT's are the dielectric properties, namely the dielectric constant e (F/m) and the dissipation factor or loss tangent *tan*d. The dielectric constant is a measure of the charge stored across a dielectric material when raised to a given voltage. The dielectric constant of a vacuum is $e_0 = 8.85x10^{-12}$ F/m. The relative dielectric constant *K* (often referred to as just "the dielectric constant") is the dimensionless ratio of e / e_0. In case of an AC field, the dielectric constant has both a real part and an imaginary part; the loss tangent *tan*d is defined as the ratio of the imaginary part to the real part. The values of these constants depend on the PZT compositions.

Resonance

Any material that has mass also has specific frequencies at which it resonates. When excited at this resonant frequency, *fr*, the material's impedance is minimum and the oscillation amplitude is at Maximum. At resonance the material will vibrate freely with greater amplitude than at other frequencies. Following this resonant frequency is an anti-resonant frequency, *fa*, where the impedance of the material is at a maximum and the oscillation amplitude is at a minimum. Piezoelectric ceramics are no different and the measurement of these characteristic frequencies provides the means to evaluate the piezoelectric and elastic properties of the ceramic.

Thickness mode, although very important in transducer design, is but one of a multitude of one dimensional resonator geometries that can be fabricated and tested. In this application note we are only considering thickness mode. Different modes of vibration, such as thickness or planar, give insight to the different constants associated with that mode. A typical resonance plot of impedance vs frequency for a piezoelectric ceramic near a resonance is shown in Figure 1.

As the frequency is increased, the oscillation first approaches a frequency at which the impedance is minimum; the resonant frequency, *fr*.

This *minimum impedance frequency*, f_m, approximates the *series resonance frequency*, f_s, and the frequency at which the impedance in an electrical circuit describing the material is zero (if resistance caused by mechanical losses is ignored).

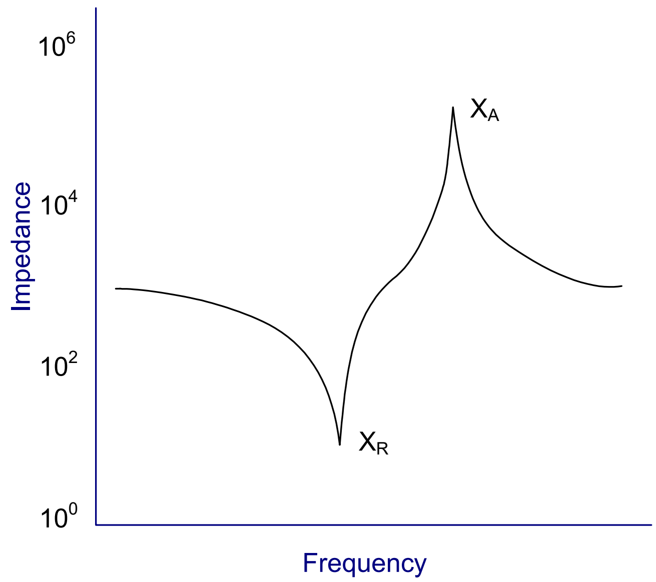


Figure 1
Resonance and Anti-Resonance

As the frequency is further increased, impedance increases to a maximum; the anti-resonant frequency, *fa*. The *maximum impedance frequency*, f_n, approximates the *parallel resonance frequency*, f_p, the frequency at which parallel resistance in the equivalent electrical circuit is infinite (if resistance caused by mechanical losses is ignored).

Maximum response from the material will be at a point between f_m and f_n. In some applications (ultrasonic motors, other high power applications) it can be advantageous to operate a piezoelectric material at its anti-resonance frequency, rather than at its resonance frequency.

At resonance, a piezoelectric element may be modeled by the equivalent circuit as shown in Figure 2. This circuit is commonly referred to as Van Dyke's Model and is recommended by the IEEE Standard on Piezoelectricity [i].

Where:
C0 is the Static Capacitance
C1 is the Dynamic Capacitance
L1 is the Dynamic Inductance
R1 is the Dynamic Resistance

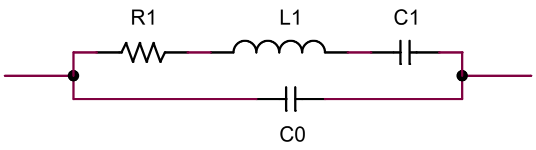


Figure 2
Van Dyke Piezoelectric Model

Below *fr* and above *fa*, the ceramic behaves like a capacitor; however between these two frequencies, the ceramic behaves like an inductor. The reader is advised to be aware; this model is only valid near and at resonance of the piezoelectric sample.

Several circuits have been used to measure *fr* and *fa* of a piezoelectric ceramic[ii]. These circuits usually consist of an oscillator for exciting the sample, a voltmeter for measuring current through the circuit, plus additional discrete components for other measurements. To find *fr*, the oscillator is varied in frequency until the maximum current is detected through the circuit. Similarly, for *fa*, the frequency of minimum current is determined.

It should be noted that there are actually six frequencies that may be identified for a particular resonance which include *fm* and *fn*; *fp* and *fs; and fr* and *fa*. IEEE Standard 177 identifies these six frequencies and establishes that for many cases, including piezoelectric ceramics, one can make the assumption that *fm* = *fs* = *fr* and *fa* = *fp* = *fn*. However, for lossy materials, such as some piezoelectric thin films, this assumption can introduce significant errors and the six frequencies should be considered separately.

Manual Piezoelectric Testing

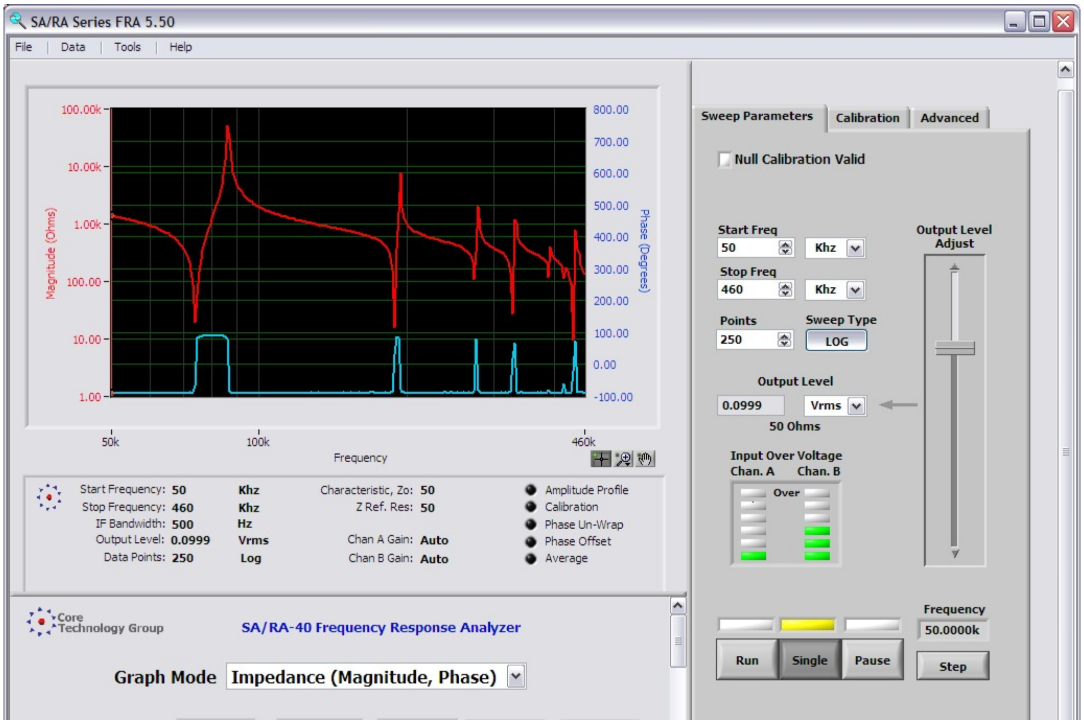
A typical manual piezoelectric test procedure is outlined below.

1. Place the ceramic material into position for measuring.
2. Adjust the frequency generator to give a maximum voltage value on the voltmeter. This value is the resonance frequency (*fr*).
3. Connect a potentiometer in series between the oscillator and voltmeter.
4. Adjust the potentiometer to provide a voltage value on the voltmeter equal to the value in Step 2. This value is the minimum impedance (Zm).
5. Remove the potentiometer.
6. Adjust the frequency generator to give a minimum voltage value on the voltmeter. This value is the anti-resonance frequency (*fa*).
7. If desired, connect the pot between the oscillator and voltmeter.
8. Adjust the pot to give a voltage value on the voltmeter equal to the value in Step 6. This value is the anti-resonance impedance (Zn).

This procedure takes time, you need to record the data manually and you have to repeat the process for all resonant frequencies you wish to measure. You measure resonant frequencies by tuning all the frequencies above and below the nominal resonant frequency and comparing the amplitude of the oscillations. In addition to the work involved, this process has the disadvantages of being difficult to perform, offers low accuracy and displays no phase information.

A relatively inexpensive SA series Vector Network Analyzer (Impedance Analyzer) from Core Technology Group was used to simplify the measurement process. This analyzer is the size of a small book and it uses a PC for the user interface. This makes it portable and especially easy to capture waveforms for documentation purposes. There are more expensive Vector Network Analyzers available which are bulky, are a lot more difficult to operate and offer few additional features,

Using a network analyzer simplifies the process because it sweeps between a frequency range and displays all the resonant frequencies at one time. Figure 3 shows a typical wide frequency sweep of a typical PZT sample.



Start Frequency: 50 Khz
Stop Frequency: 460 Khz
IF Bandwidth: 500 Hz
Output Level: 0.0999 Vrms
Data Points: 250 Log

Characteristic, Zo: 50
Z Ref. Res: 50
Chan A Gain: Auto
Chan B Gain: Auto

● Amplitude Profile
● Calibration
● Phase Un-Wrap
● Phase Offset
● Average

Graph Mode: Impedance (Magnitude, Phase)

☐ Null Calibration Valid

Start Freq: 50 Khz
Stop Freq: 460 Khz
Points: 250
Sweep Type: LOG
Output Level: 0.0999 Vrms
50 Ohms
Frequency: 50.0000k

Figure 3
Network Analyzer Screen

The values for minimum impedance frequency and maximum impedance frequency can be used to calculate the *electromechanical coupling factor*, k, an indicator of the effectiveness with which a piezoelectric material converts electrical energy into mechanical energy or mechanical energy into electrical energy. The value of, k depends on the mode of vibration and the shape of the ceramic material. Dielectric losses and mechanical losses also affect the

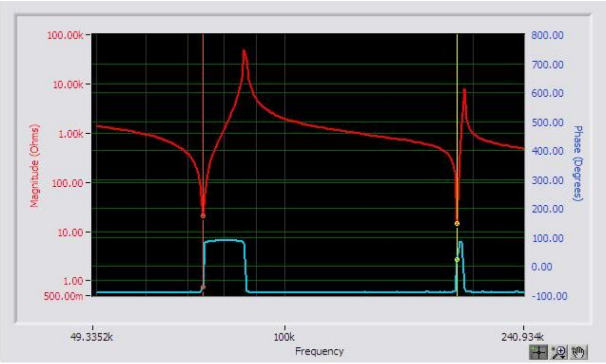


Figure 4
Resonance Close-up

efficiency of energy conversion. Dielectric losses are usually more significant than mechanical losses.

The network analyzer allows the user to zoom in to a specific frequency range to extract details needed to complete the calculations. In figure 4, we see two resonance points in the top waveform and their phase in the lower waveform.

The equation for the thickness mode (IEEE Standards ANSIIEEE 176 -1987) with complex coefficients is:

$$Z = \frac{l}{i\omega A\varepsilon_{33}^{S}}\left(1 - \frac{k_t^2 \tan\left(\frac{\omega}{4f_P}\right)}{\frac{\omega}{4f_P}}\right) \qquad (2)$$

In the above equation $\varepsilon_{33}^{S}, f_P, k_t$ are complex and produce an impedance spectrum as a function of frequency $Z(\omega) = R(\omega) + iX(\omega)$ or $Y(\omega) = G(\omega) + iB(\omega)$ (Admittance) that is also complex. It is beneficial to look at the effect of the real and imaginary components of the material coefficients on the impedance spectrum. Figure 5 shows the Real and Imaginary Impedances at the resonant frequency *fr* and the impedance *Xr* of a thickness piezoelectric sample. It is a simple matter to convert the frequency *fr* to angular frequency ω_r by $\omega_r = 2\pi f_r$.

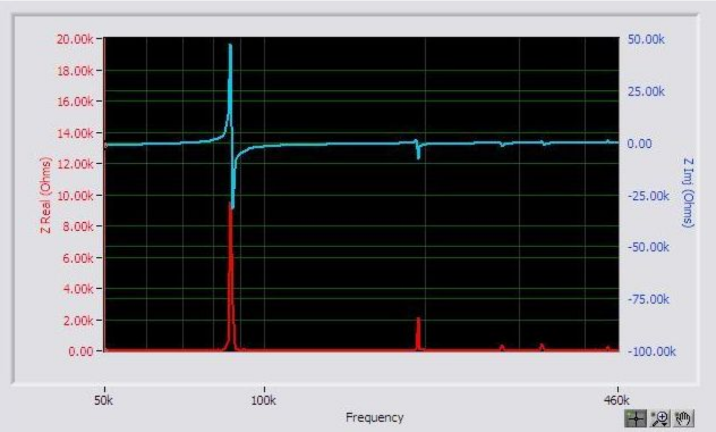


Figure 5
Real and Imaginary Impedances

The complex parallel resonance frequency $\boldsymbol{f_\rho}$ (maximum in resistance) is a function of the complex stiffness c_{33}^{D} and the density ρ through the equation:

$$c_{33}^{D} = 4\rho l^2 f_\rho^2 \tag{3}$$

The real value of the stiffness is therefore equal to the real part (Red) of f_ρ while the imaginary part (Blue) of f_ρ is equal to the half width (130 Hz) at half maximum about f_ρ (50K Ohms). The final coefficient that can be determined from the spectra is the electro-mechanical coupling coefficient k_t. The real part of the coupling is proportional to the difference of the real part of f_ρ and the real part of f_s the series resonance frequency (maximum in G). The imaginary part of the coupling is proportional to the ratio of the imaginary part of f_ρ and the imaginary part of f_s which is a measure of the change in the half width at half maximum at the parallel and series resonance. If the imaginary component of the coupling is zero then by definition $\Delta f_s / f_s = \Delta f_p / f_p$ and the breadth of the resonance in each spectrum is the same.

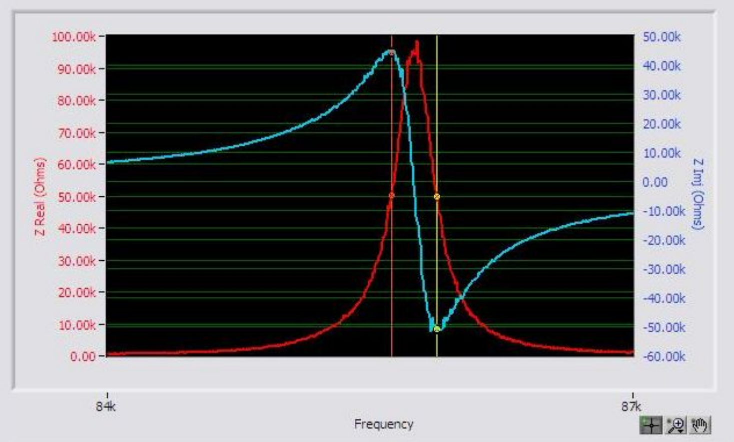


Figure 6
Close-up Showing f_ρ

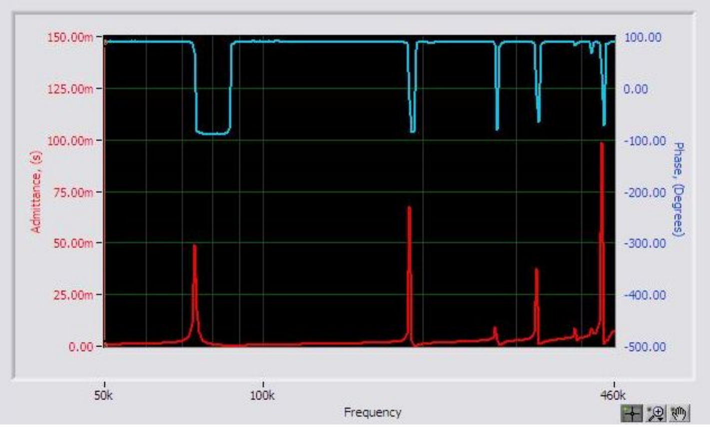
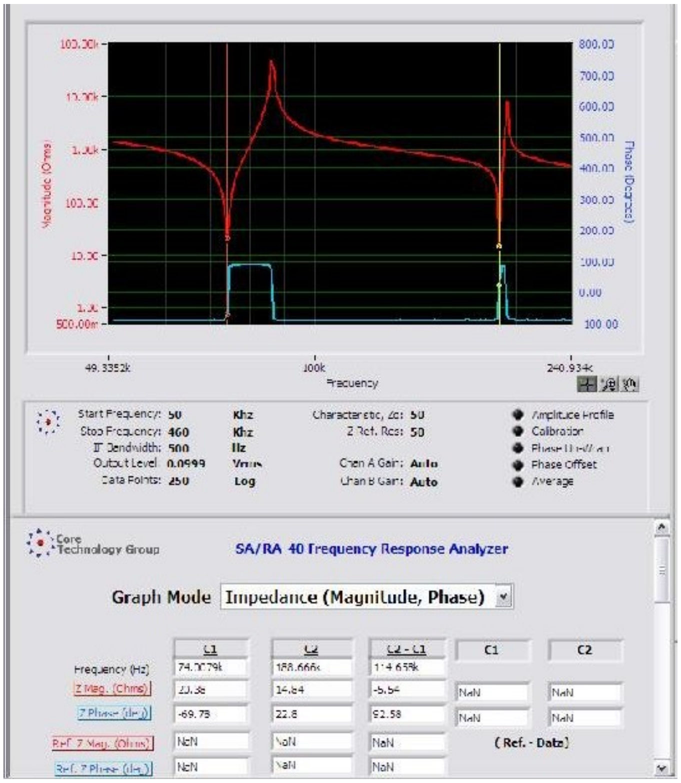


Figure 7
Admittance and Phase

The network analyzer display shown in Figure 7 also allows the user to look at the Admittance ($Y(\omega)$) at different frequencies, by simply selecting a different display option. The relative position of the waveforms does not change making it simple to see which resonant frequencies have the highest Admittance.

By positioning cursors over the waveform, as shown in Figure 8, the lower portion of the screen shows some data needed for piezoelectric calculations. In this case, Impedance and Phase at two resonant frequencies is shown. There is no need to measure the resistance of a potentiometer to get this information. And, it only took 20 seconds to capture all this data using a Network Analyzer.

Another thing the network analyzer does is make it easy to qualify or "Bin" piezoelectric samples. By saving an "Ideal" waveform, it becomes a simple matter to test a batch of piezoelectric samples and select those samples that come close to the "Ideal". This could be useful when stacking piezoelectric elements to match resonant nodes.



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Data Points: 250 Log
Characteristic, Zc: 50
Z Ref. Res: 50
Chan A Gain: Auto
Chan B Gain: Auto

Core Technology Group
SA/RA 40 Frequency Response Analyzer

Graph Mode: Impedance (Magnitude, Phase)

	C1	C2	C2 - C1	C1	C2
Frequency (Hz)	74.0079k	188.666k	114.658k		
Z Mag. (Ohms)	20.38	14.84	-5.54	NaN	NaN
Z Phase (deg)	-69.73	22.8	92.58	NaN	NaN
Ref. Z Mag. (Ohms)	NaN	NaN	NaN	(Ref. - Data)	
Ref. Z Phase (deg)	NaN	NaN	NaN		

Figure 8
Data Displayed Using Cursors

Resonance Measurement Considerations

Resonance analysis is the simplest method to determine the small signal elastic, dielectric, and piezoelectric properties of a sample, however there are a variety of points that should be considered if you want to determine accurate, reliable and repeatable material properties:

a) Know the limits of your instrument. During the sweep of a resonance spectrum, the impedance of the piezoelectric sample can change over 6 orders of magnitude between the resonance and anti-resonance frequencies. In order to determine material coefficients accurately, especially the loss terms, make sure the instrument can accurately measure the impedance over this range. If doing this manually, make sure your analysis does not include data from regions which may not have been measured accurately.

b) If the instruments have open and short circuit corrections for the test-probe or probe compensation these should always be used. This is especially true when custom built test holders are used.
c) The recommended sample aspect ratios must be used to ensure that you have an isolated resonance mode. In the case of extremely anisotropic material properties, these recommended aspect ratios may have to be increased.
d) As far as possible the leads to the electrodes of the resonator should be attached at nodal points and, if the contact is not a permanent solder contact, the contact force must be small and the mass of the contact should not be significant. The sample should also be mounted in a symmetric manner with respect to the mode.
e) For thin large lateral dimension resonators such as the thickness, radial and length thickness mode resonators the sheet resistance of the electrodes should be much less than the resistance at resonance.

Summary

Characterization of the elastic, dielectric and electromechanical properties of piezoelectric ceramics is crucial for several reasons.

First, investigations of the material properties provide a link between the manufacturing process and ceramic performance. This enables the developer of the materials to adjust the manufacturing process of the ceramic to produce tailored materials.

Second, the engineer can investigate prospective materials for applicability to a specific need.

Third, material parameters obtained through characterization can be used to develop and validate analytical models of the ceramics.

The Network Analyzer is an essential tool in characterizing piezoelectric ceramics. This tool speeds up the testing process while making data collection easier and more accurate. Insights gained through characterization have led to many new devices and uses. For example, investigation of the hydrostatic coefficients of PZT and those of the piezoelectric polymer polyvinylidene fluoride, PVDF, identified the product of *dh* and *gh* as a figure of merit and led to combining both materials yielding a superior device that better fits hydrophone applications.

More than a century after their discovery, piezoelectric ceramics have become viable for commercial purposes. Researchers continue to diligently uncover novel ways to characterize the complex electromechanical properties and the network analyzer is fast becoming a valuable tool in this research.

Bibliography

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